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Avoiding Unintended Consequences:

How Incentives Aid Information Provisioning in Bayesian Congestion Games

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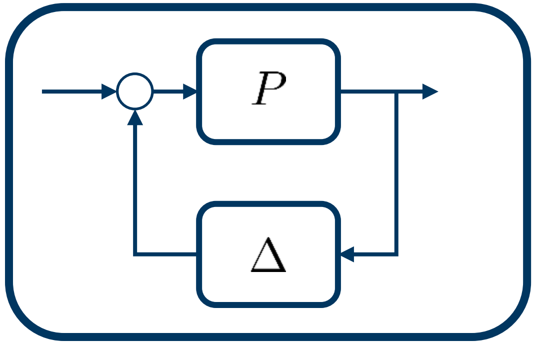
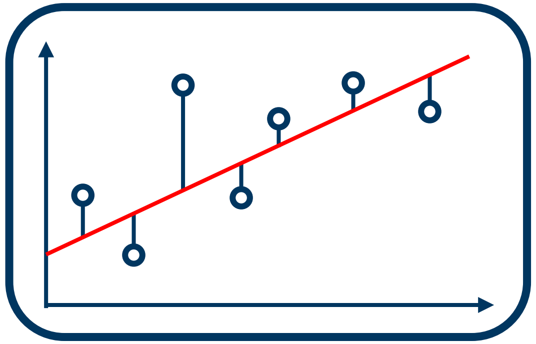
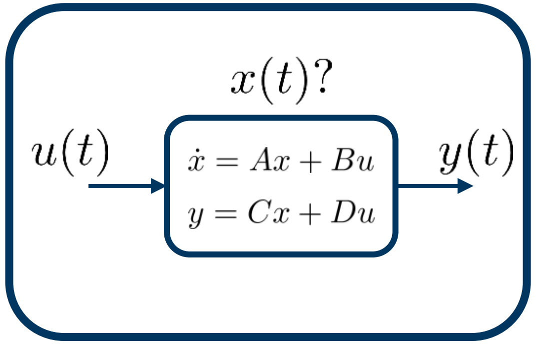
Information in Control



Robust Control


Estimation



Observability

Classic Problem:

Uninformed designer must ***combat*** uncertainty ***in*** control

Flip Information Paradigm

Emerging Problem:

Informed designer can ***utilize*** uncertainty ***as*** control

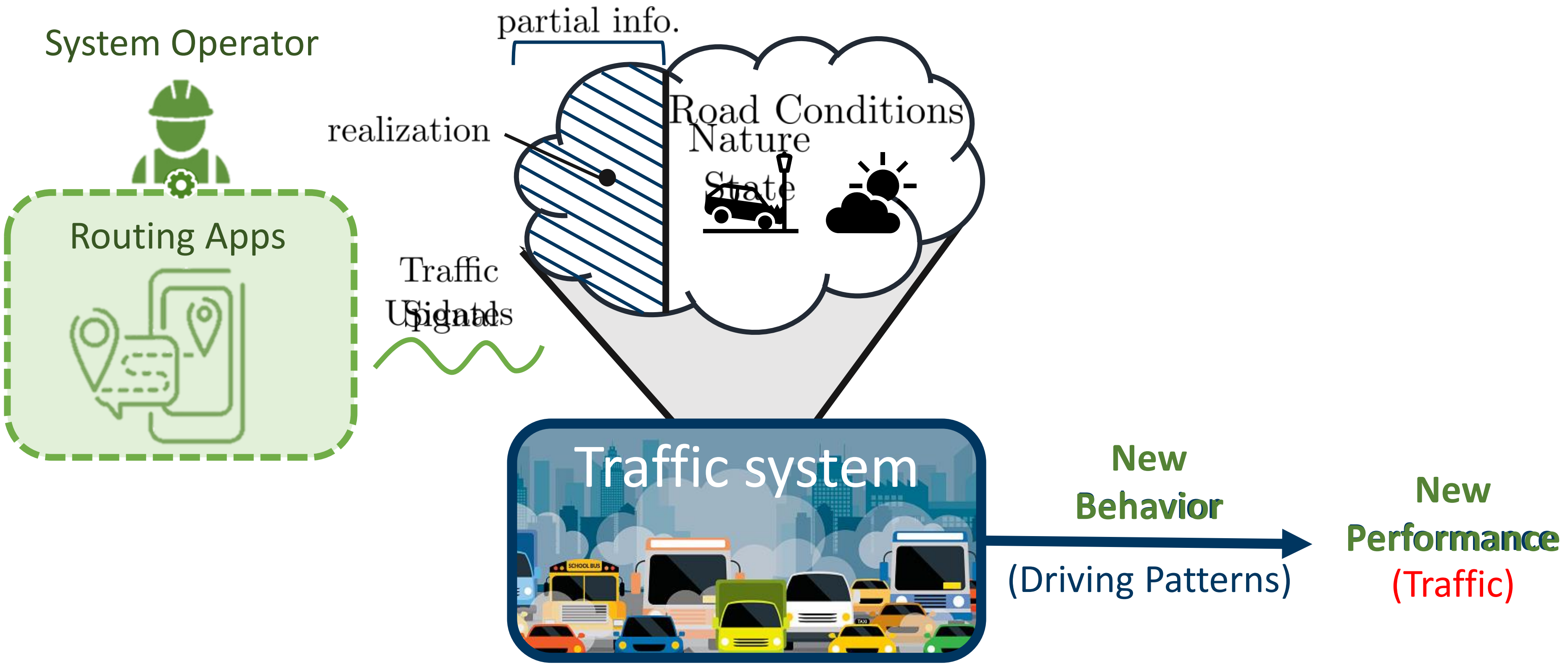
Information *as* Control

Information Signalling

Operator observes realization of state

↓

Partially signal information to agents

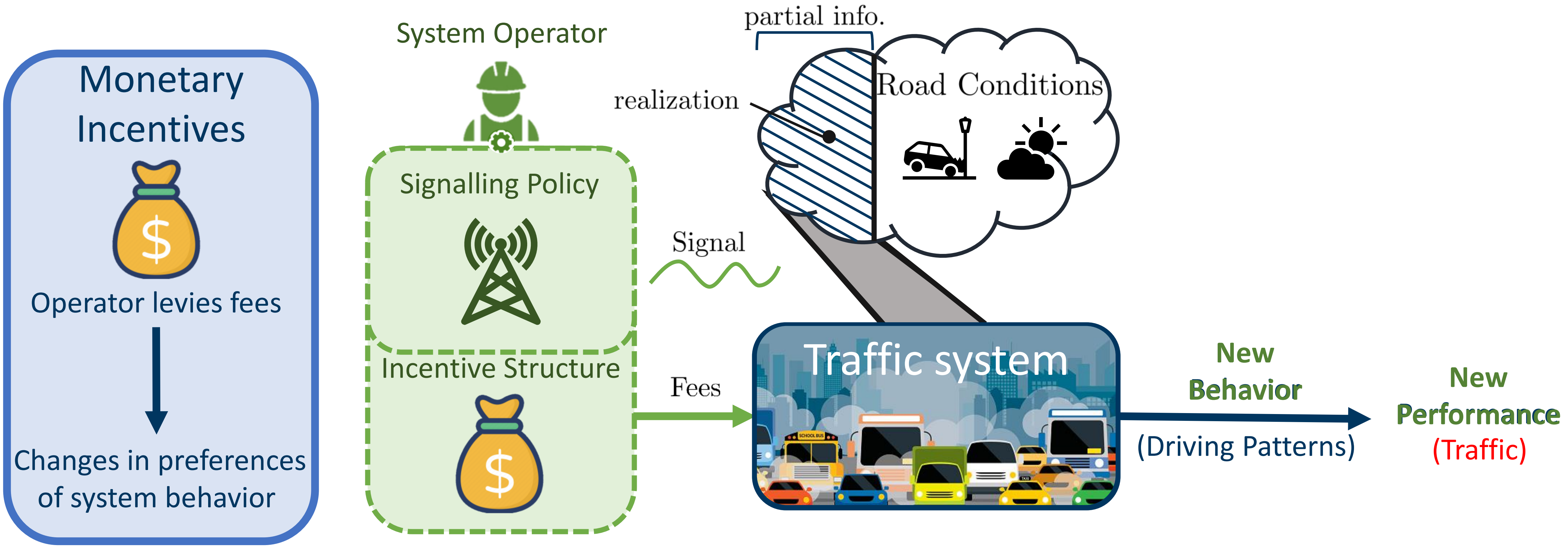


Q?: Does signalling improve performance?

Theorem 1

Signalling can **help** *or* **hurt**

Information & Incentives



Q?: How do *signals* and *incentives* work together?

Theorem 2
With incentives
Signalling can ***only*** help

Important Questions

Today's focus

- ***When*** should information be revealed?
- How much can we ***improve*** performance?
- How do ***incentives*** and signals interact?

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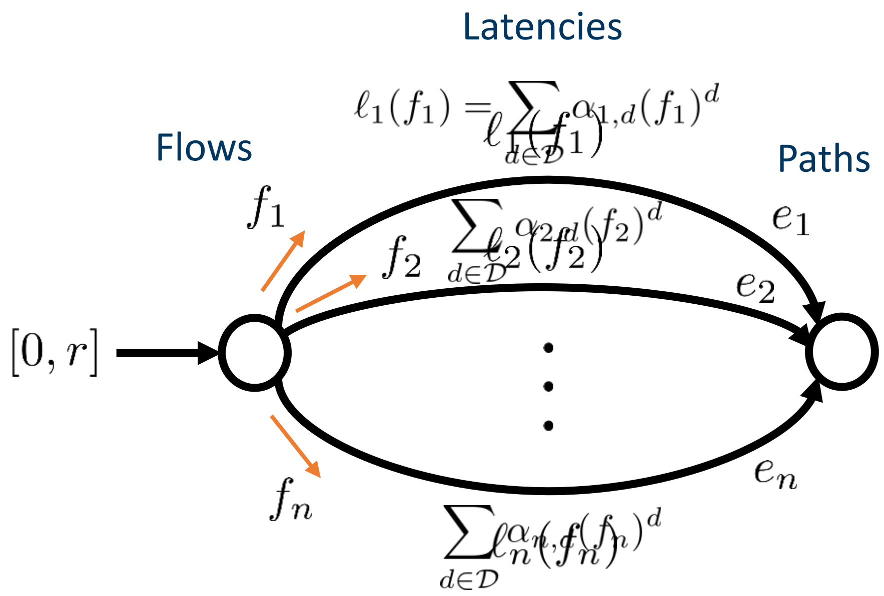
(submitted)

- How do we ***design*** signalling policies?
- Can ***deceit*** help us?

In Progress

Bayesian Congestion Games

Full information



Latency function:

$$\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d}(f_e)^d$$

Possible degrees

$$\mathcal{D} = (d_1, \dots, d_k)$$

Affine $\mathcal{D} = \{0, 1\}$

BPR $\mathcal{D} = \{0, 4\}$

Polynomial $\mathcal{D} = \{0, 1, \dots, D\}$

Nash flow/Wardrop Equilibrium: f^{Nf}

$$\ell_e(f_e^{\mathrm{Nf}}) \leq \ell_{e'}(f_{e'}^{\mathrm{Nf}}), \ \forall e \in E \text{ s.t. } f_e^{\mathrm{Nf}} > 0, \ e' \in E$$

Each user takes minimal cost path

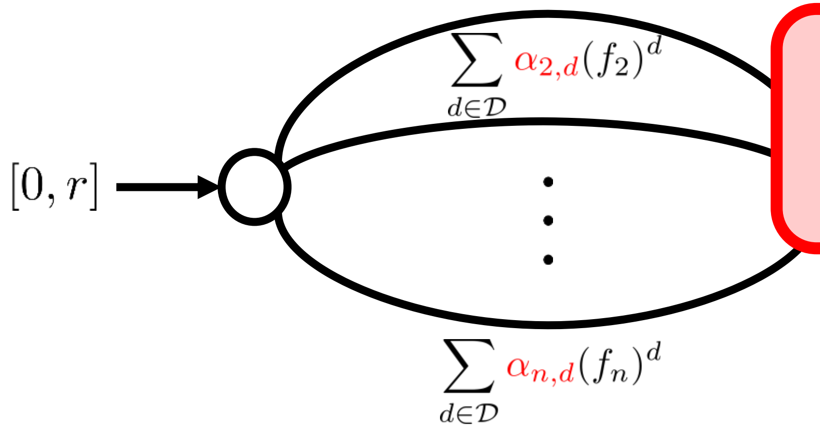
System Cost: Total Latency

$$\mathcal{L}(f) = \sum_{e \in E} f_e \cdot \ell_e(f_e)$$

Bayesian Congestion Games

Unknown State → Uncertain Congestion Rates: $\alpha = \begin{bmatrix} \vdots \\ \alpha_{e,d} \\ \vdots \end{bmatrix} \in \mathbb{R}^{n \cdot |\mathcal{D}|}_{\geq 0}$

$\ell_1(f_1) = \sum_{d \in \mathcal{D}} \alpha_{1,d}(f_1)^d$



Latency function:

$$\ell_e(f_e) = \sum_{d \in \mathcal{D}} \alpha_{e,d}(f_e)^d$$

Possible degrees

$$\mathcal{D} = (d_1, \ldots, d_k)$$

Prior belief:

$x]$

How *effective* is signalling in reducing system cost?

$) \to (\text{signal})$

$\pi = \{\pi_1, \ldots, \pi_m\}$ public and truthful

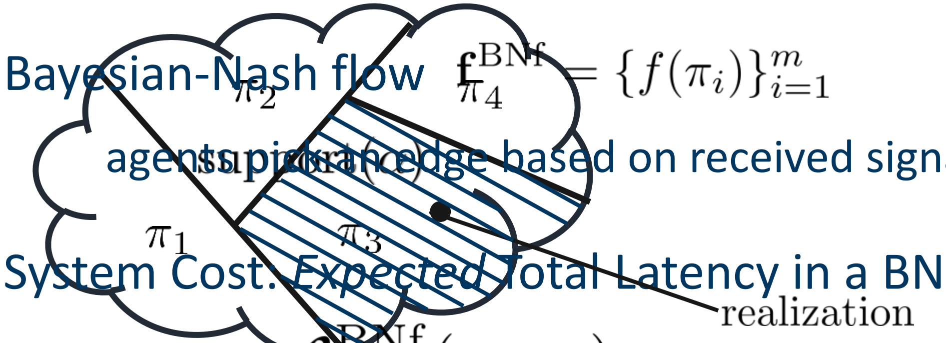
Posterior belief

$$\mu_i(x) = \mathbb{P}[\alpha = x | \alpha \in \pi_i]$$

Bayesian-Nash flow $\mathbf{f}^{\text{BNf}}_{\pi} = \{f(\pi_i)\}_{i=1}^m$

agents pick an edge based on received signal

System Cost: *Expected* Total Latency in a BNf



$$\mathcal{L}^{\text{BNf}}(\pi; \mu_0)$$

Benefit of Signalling

Performance metric: *Benefit of Signalling*

$$\mathbf{B}(\pi;\mu_0) = \underbrace{\mathcal{L}^{\mathrm{BNf}}(\emptyset;\mu_0)}_{\text{without signalling}} - \underbrace{\mathcal{L}^{\mathrm{BNf}}(\pi;\mu_0)}_{\text{with signalling}}$$

Reduction in system cost from signalling

Theorem 1: For any set of polynomial degrees $\mathcal{D}$, prior μ_0, and signalling policy π:

$$-\sqrt{|\mathcal{D}|}\cdot\|\mathbb{E}[\alpha]-\underline{\alpha}\|_2 \leq \mathbf{B}(\pi;\mu_0) \leq \sqrt{|\mathcal{D}|}\cdot\|\mathbb{E}[\alpha]-\underline{\alpha}\|_2,$$

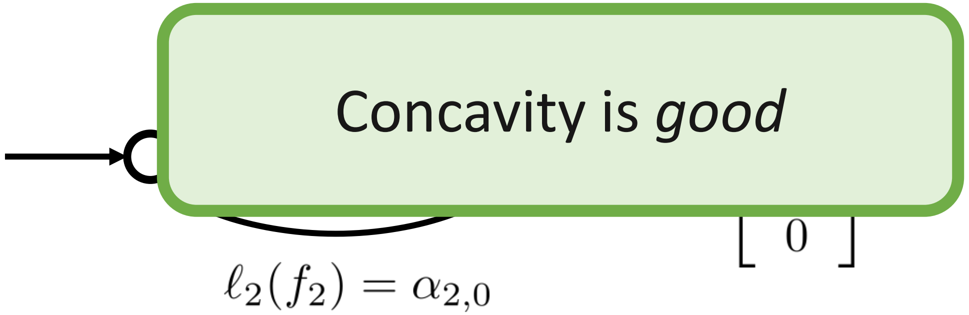
where $\mathbb{E}[\alpha] = \int_{x\in A} x\cdot\mu_0(x)dx$, and $\underline{\alpha} \in \mathbb{R}_{\geq 0}^{|E|\cdot|\mathcal{D}|}$ such that $\underline{\alpha}_{e,d} = \inf\{\mathrm{supp}(\alpha_{e,d})\}$ for each $e \in E$, $d \in \mathcal{D}$.

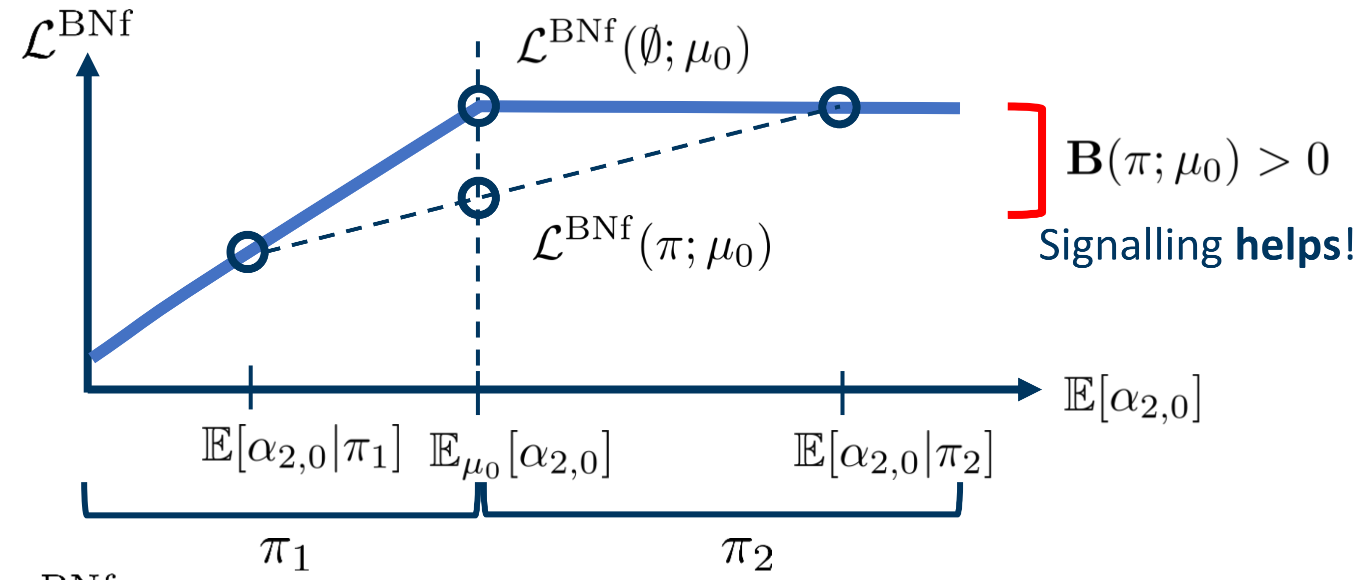
Observations:

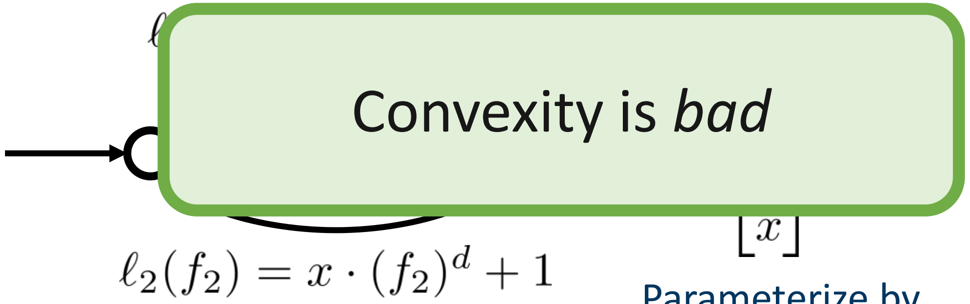
1. Signals can help *or* hurt performance
2. Bounds depend on
   I. Complexity of model
   II. Spread of α

Insights and Reasoning

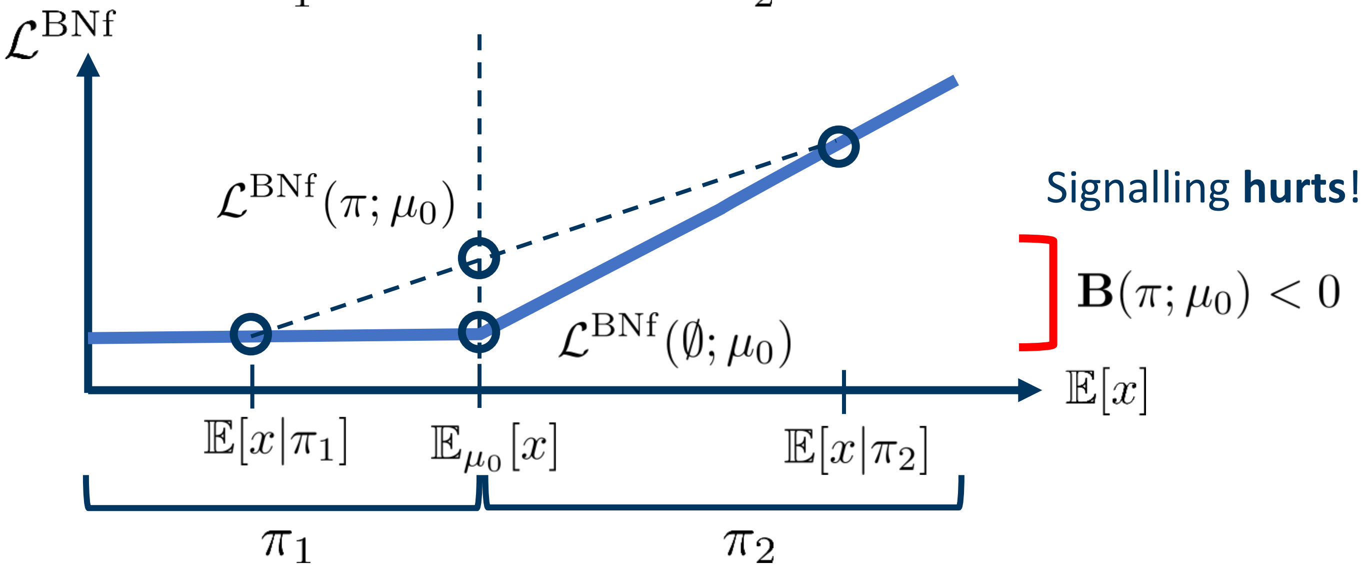
Lemma: $\mathcal{L}^{\text{BNf}}$ depends only on $\mathbb{E}[\alpha|\pi_i]$





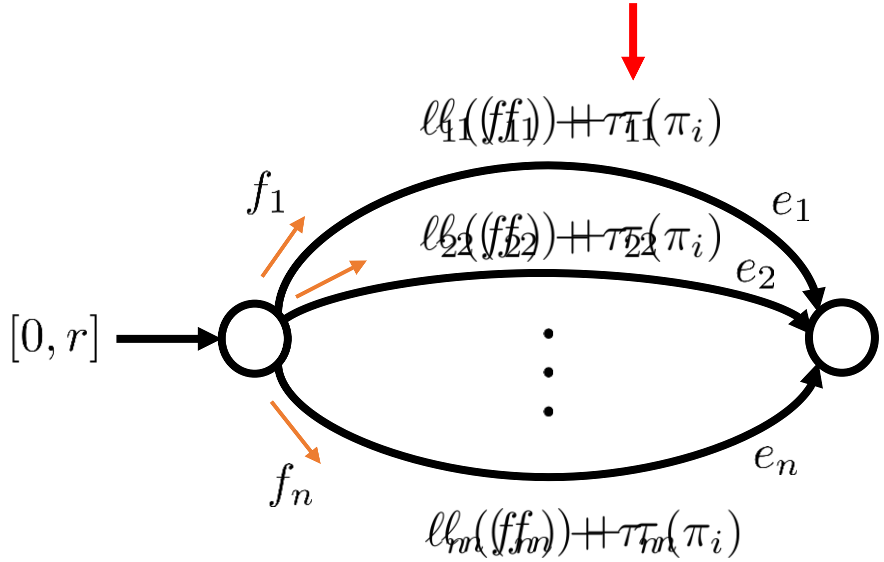


Parameterize by single variable x



Monetary Incentives

Add a monetary incentive τ_e to each edge to alter costs



Signal Aware Incentives:

$\tau_e(\pi_i)$

Proposition: For a signalling policy π, the optimal signal aware incentive is

$$\tau_e^\star(\pi_i) = \sum_{d \in \mathcal{D}} d \cdot \mathbb{E}[\alpha_{e,d} | \pi_i] z_e^d$$

where $z \in \arg\min_f \mathcal{L}\left(f; \mathbb{E}[\alpha|\pi_i]\right)$.

Concurrent Signals and Incentives

With incentives, what is the benefit of signalling? $\mathbf{B}(\pi; \mu_0, \tau^\star)$

Theorem 2: For any set of polynomial degrees $\mathcal{D}$, prior μ_0, and signalling policy π:

$$0 \leq \mathbf{B}(\pi; \mu_0, \tau^\star) \leq \sqrt{|\mathcal{D}|} \cdot \|\mathbb{E}[\alpha] - \underline{\alpha}\|_2,$$

where $\mathbb{E}[\alpha] = \int_{x \in A} x \cdot \mu_0(x) dx$, and $\underline{\alpha} \in \mathbb{R}_{\geq 0}^{|E| \cdot |\mathcal{D}|}$ such that $\underline{\alpha}_{e,d} = \inf\{\text{supp}(\alpha_{e,d})\}$ for each $e \in E$, $d \in \mathcal{D}$.

Observations:

1. With incentives, signalling *can only help*
2. Signalling still has the same capabilities to improve performance

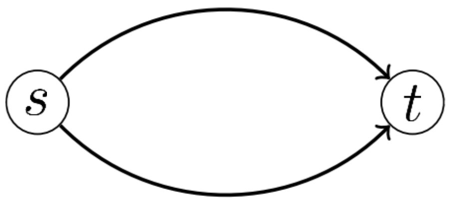
Numerical Example

Benefit of Signalling

Theorem 1
Without incentives
Signalling can help *or* hurt

Theorem 2
With incentives
Signalling can *only* help

$$\ell_1(f_1) = (f_1)^2 + \alpha_{1,0}$$

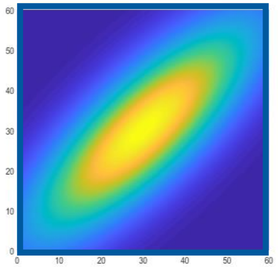


$$\ell_2(f_2) = \alpha_{2,2}(f_2)^2 + 1$$

$$\alpha = \begin{bmatrix} \alpha_{1,0} \\ 1 \\ 1 \\ \alpha_{2,2} \end{bmatrix}$$

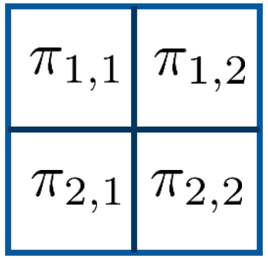
prior μ_0

$\alpha_{1,0}$

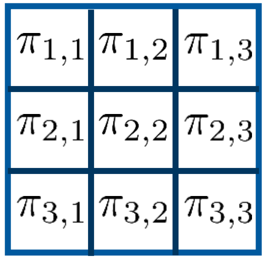


$\alpha_{2,2}$

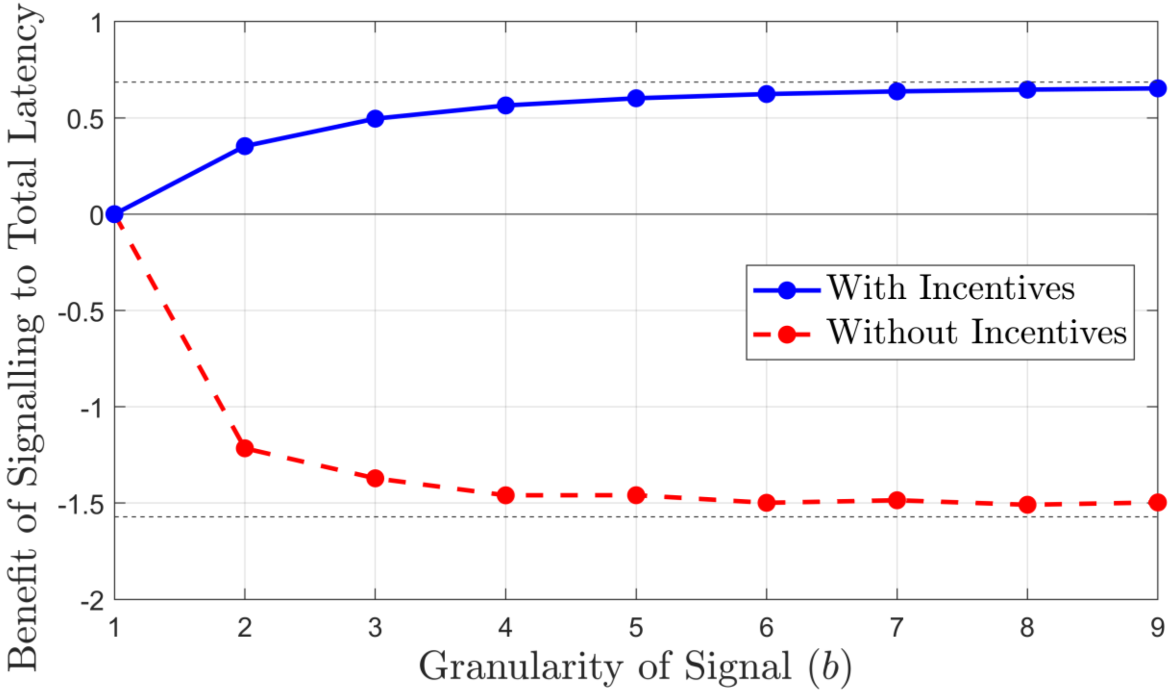
$b = 1 \Rightarrow \pi = \emptyset$



$b = 2$



$b = 3$



Conclusions

- ***When*** should information be revealed?

When you have ***concavity***
When you use ***incentives***

- How much can we ***improve*** performance?

Bounds on benefit
Similar improvement when using incentives

- How do ***incentives*** and signals interact?

Incentives essentially ***robustify*** signals

- How do we ***design*** signalling policies?

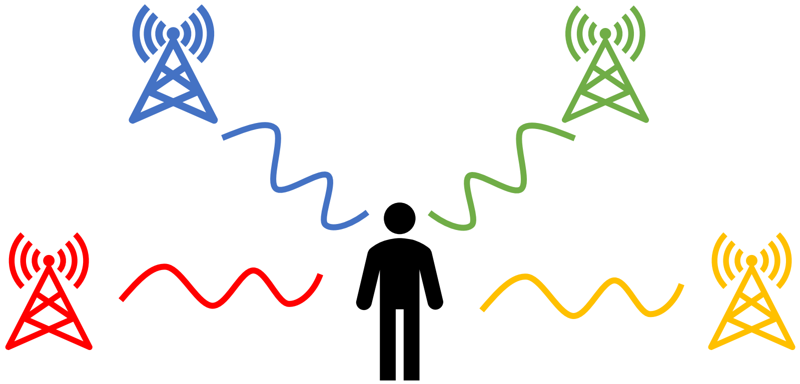
Hard because decision variable is any partition
Limit to certain types of signals

- Can ***deceit*** help us?

Conjecture: Not with public signals

Future Directions

- Multiple information senders



- Non-Bayesian Inference

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